# The Theory of the Nucleon Spin

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I discuss two topics of current interest in the study of the spin structure of the nucleon. First, I discuss whether there is a sum rule for the components of the nucleon's angular moments. Second, I discuss the measurement of the nucleon's transversity distribution in light of recent results reported by the HERMES collaboration at DESY.

**Keywords: transversity, QCD, spin, sum rules**

## 1. Introduction

Quantum chromodynamics is the only nontrivial quantum field theory that we are certain describes the real world. The electroweak part of the Standard Model, despite its richness, is adequately described by perturbation theory. String theory and quantum gravity, despite their promise, remain far from experimental validation. Perturbative methods are applicable to QCD at short distances, where they confirm that it is the right theory of the strong interactions. However at $10^{-13}$ cm, where QCD phenomena are rich and complex, perturbative methods are hopeless. Even a simple scattering process like $\pi N \rightarrow \pi\pi N$ at $E_{\rm cm} = 2$ GeV and moderate momentum transfer seems far beyond our abilities: the energy is too low for perturbative methods, but too high for chiral dynamics; the process is dominated by broad overlapping and interfering resonances in the $\pi N$ and $\pi\pi$ channels; existing lattice methods are useless.

Perhaps QCD will never be solved at this level of detail. On the other hand, perhaps it is not important to know every hadron–hadron scattering amplitude. Where, then, is it important and interesting to understand QCD? Each of us has his or her own prejudices. The organizers, judging from the program, favor the deep inelastic domain, where there are two obviously important reasons to study QCD. First, this is where perturbation theory applies so we can test whether QCD is the correct theory of the strong interactions. It has survived all tests to date, but discovery of even a small deviation would be extremely exciting. Second, we need to understand parton distribution and fragmentation functions in order to use hadron colliders to search for new physics. "QCD Engineering" as it might be called, is an essential ingredient in planning the development and exploitation of the Tevatron and LHC.

Many of us believe there is another important regime where QCD merits intense study: in the domain of its lowest states, where symmetries and regularities abound, some of them only poorly understood. Examples include the Quark Model, SU(3)-flavor symmetry, the OZI rule, and vector dominance. The Quark Model, for example, assumes—for no good reason—that quark number is conserved in the spectrum of hadrons. Thus the nucleon is a $Q^3$ state and the pion is $\bar{Q}Q$. Huge amounts of spectroscopic data can be cataloged using the Quark Model and SU(3) flavor symmetry. However, when we look closely at hadrons—in deep inelastic scattering experiments, or in relativistic, field-theoretic models—we find that the nucleon is full of gluons and $\bar{Q}Q$ pairs, and the pion is often better regarded as a coherent wave on the chiral condensate as opposed to a $\bar{Q}Q$ state. Why, then, does the simple Quark Model work so well? The answer must lie in the confinement dynamics of QCD. This is an old problem, but the wealth of available data makes low energy QCD in general, and the quark structure of hadrons in particular, a tantalizing playground for a theorist with a new idea about confinement.

There is no uniform framework for theoretical studies of the nucleon in QCD. We have no relativistic analogue of the Schrödinger wavefunction, which would summarize all that can be asked about the nucleon in the way it does for the states of the hydrogen atom. The best we have at the moment seems to be a list of the expectation values of local operators in the nucleon state, $\langle\Theta\rangle_N$, and the generalization of this concept to the parton distribution function, $\langle\Theta(x, Q^2)\rangle_N$. Table 1 shows a schematic list of local operators. In the middle are operators without derivatives

| | |
|---|---|
| Moments of parton distributions $\uparrow$ | $\bar{q}\gamma^+D^+\cdots D^+q$ |
| | $\vdots$ |
| | $\bar{q}\gamma^+D^+q$ |
| | $\vdots$ |
| | $q^\dagger q,\ \bar{q}\vec{\gamma}\gamma_5 q,\ \bar{q}mq,\ \bar{q}\sigma^{0i}\gamma_5 q$ |
| | $\vdots$ |
| Derivatives of form factors $\downarrow$ | $q^\dagger \frac{1}{2}\vec{r}\times\vec{\sigma}q$ |
| | $\vdots$ |
| | $q^\dagger r^2 q$ |

Table 1.

in either coordinate or momentum space. Upwards, are operators with coordinate space derivatives. The simplest ones, those involving derivatives along the light cone, are given by the moments of parton distributions, $\int dx\, x^n \langle\Theta(x,Q^2)\rangle_N$. Others, with transverse or $x^-$ derivatives are more complicated ("higher twist") and harder to interpret. Downwards, are operators with insertions of the coordinate $x^\mu$—moments of local operators—equivalent to derivatives in momentum space. The classic examples are the magnetic moment and the mean-square charge radius. These two can be related to the derivatives of elastic form factors at zero-momentum transfer. Higher moments can be defined, but they *are not* related to form factors in a simple way.

A theorist interested in precise and interpretable information about the nucleon is limited to the expectation values of these few local operators. Of course, changing the Dirac and flavor structure and generalizing to gluonic operators, makes for quite an extensive list and quite a challenge to our experimentalist colleagues. But it is far less than the content of the Schrödinger wavefunction. The upward extrapolation of the list in Table 1 leads to the notion of a parton distribution function. These give us our most detailed, interpretable, and accessible description of the nucleon. At least at leading twist, parton distribution functions have great heuristic value, providing a snapshot of the momentum and helicity weighted distributions of quarks and gluons in a rapidly moving nucleon. The "momentum" variable is Bjorken's $x$, which is identified with $Q^2/2P\cdot Q$ in deep inelastic scattering and with the parton's momentum fraction in an infinite momentum frame. Attempts to generalize this intuitive picture beyond leading twist and to fragmentation functions have been only partially successful.

My talk will concern a few topics at the interface between deep inelastic scattering and hadron structure, where our understanding of perturbative QCD is so good that we can hope to use it to obtain new information about the strongly coupled domain where confinement operates. Specifically, I want to review what

is known about the measurement of the components of the nucleon's angular momentum and its transversity. The situation with angular momentum is not very satisfactory: we know how to define the $x$-distributions of the various components of the nucleon's angular momentum, but we do not know how to measure them. On the other hand, there is now great excitement about transversity: Hermes has reported an azimuthal asymmetry in single particle inclusive deep inelastic scattering, which, if confirmed, would suggest a large analyzing power for transversity in a very accessible experiment.

## 2. Is there an "Angular Momentum Sum Rule" and is it experimentally testable?

The answers to the questions posed in the title to this section are "Yes" and "Apparently, No", respectively. First, I wans to clarify some terminology. I would like to distinguish between a sum rule and an operator relation. A *sum rule* expresses the expectation value of a local operator in a state as an *integral* (or sum) over a distribution measured in an inelastic production process involving the same state. This is the traditional definition of a sum rule, dating back to the Thomas, Reiche, Kuhn Sum Rule of atomic spectroscopy. All the familiar sum rules of deep inelastic scattering—Bjorken's, Gross & Llewellyn-Smith's, etc.—are this type of relation. They are even more powerful because the distribution which is integrated has a simple, heuristic interpretation as the momentum (Bjorken-$x$) distribution of the observable associated with the local operator. The "spin sum rule" gives a typical example:

$$\begin{aligned}\langle P,S|\; \bar{q}_a\gamma^\mu\gamma_5 q_a\big|_{Q^2}\, |P,S\rangle/S^\mu \quad &\equiv \quad \Delta q_a(Q^2) \\ &= \quad \int_0^1 dx\; \left\{ q_{a\uparrow}(x,Q^2) + \bar{q}_{a\uparrow}(x,Q^2) \right. \\ &\qquad\qquad \left. - q_{a\downarrow}(x,Q^2) - \bar{q}_{a\downarrow}(x,Q^2) \right\} \qquad (2.1)\end{aligned}$$

The left-hand side can be measured in $\beta$-decay or other electroweak processes. The right-hand side can be measured in deep inelastic scattering of polarized leptons from polarized targets. The meaning of the sum rule is clear because the local operator, $\bar{q}_a\gamma^\mu\gamma_5 q_a$ is the generator of the internal rotations (the "spin") of the quark field in QCD. The sum rule says the quark's contribution to the nucleon's spin is the integral over a spin weighted momentum distribution of the quarks.

Another, less powerful but still interesting type of relation—sometimes called a sum rule in the QCD literature—arises simply because an operator can be written as the sum of two (or more) other operators, $\Theta = \Theta_1 + \Theta_2$. If the expectation values of all three operators can be measured, then this relation, and the assumptions underlying it, can be tested. Such a relation exists for the contributions to the nucleon's angular momentum (Jaffe & Manohar 1990,Ji 1997),

$$\tfrac{1}{2} = \hat{L}_q + \tfrac{1}{2}\Sigma + \hat{J}_g \qquad (2.2)$$

where the three terms are *roughly* the quark orbital angular momentum, the quark spin, and the total angular momentum on the gluons. Ji has shown how, in principal, to measure the various terms in this relation (Ji 1997).

A sum rule of the classic type also exists for the contributions to the nucleon's angular momentum (Bashinsky & Jaffe 1998, Hagler & Schafer 1998, Harindranath & Kundu 1999),

$$\tfrac{1}{2} = \int_0^1 dx\ \left\{ L_q(x,Q^2) + \tfrac{1}{2}\Delta q(x,Q^2) + L_g(x,Q^2) + \Delta G(x,Q^2) \right\} \tag{2.3}$$

where the four terms are *precisely* the $x$-distributions of the quark orbital angular momentum, quark spin, gluon orbital angular momentum, and gluon spin. However it appears that the distributions $L_q(x,Q^2)$ and $L_g(x,Q^2)$ are not experimentally accessible. So the value of the sum rule is obscure.

Before exploring these relations for the angular momentum in more depth, let's examine the simpler and well-understood case of energy and momentum.

### (*a*) *Sum rules for energy and momentum*

One hears a lot about the "momentum sum rule" in QCD, but nothing about an "energy sum rule". The reasons are quite instructive. Energy and momentum are described by the rank two, symmetric energy-momentum tensor, $T^{\mu\nu}$,

$$T^{\mu\nu} = \frac{i}{4}\bar{q}(\gamma^\mu D^\nu + \gamma^\nu D^\mu)q + \text{h.c.} + \text{Tr}(F^{\mu\alpha}F_\alpha^\nu - \tfrac{1}{4}g^{\mu\nu}F^2)\,, \tag{2.4}$$

where $D^\mu$ and $F^{\mu\nu}$ are the gauge covariant derivative and gluon field strength, both matrices in the fundamental representation of SU(3).†

The energy density is given by $T^{00}$,

$$\mathcal{E} \equiv T^{00} = \tfrac{1}{2}q^\dagger(-i\vec{\alpha}\cdot\vec{D} + \beta m)q + \text{h.c.} + \text{Tr}(\vec{E}^2 + \vec{B}^2)\,. \tag{2.5}$$

The expectation value of $T^{00}$ is normalized,

$$\langle P|T^{00}|P\rangle = 2E^2\,, \tag{2.6}$$

because $|P\rangle$ is an eigenstate of the Hamiltonian, $\int d^3x T^{00}(x)|P\rangle = E|P\rangle$. This is a good start towards a sum rule. However there is no useful sum rule because there is no way to write any of the terms in equation (2.5) as an integral over inelastic production data. This is not obvious, but the appearance of terms in $\mathcal{E}$ which are order cubic and higher in the canonical fields is a bad sign. The parton distributions of deep inelastic scattering (DIS) come from operators quadratic in the "good" light cone components of the quark and gluon fields, $q_+$ and $\vec{A}_\perp$ (Jaffe 1996). The first term in $\mathcal{E}$ includes $\bar{q}qg$ coupling, and $\vec{E}^2 + \vec{B}^2$ involves terms cubic and quartic in the gluon vector potentials $\vec{A}_\perp$.

In contrast there is a classic, deep inelastic sum rule for $P^+$, where $P^+ = \frac{1}{\sqrt{2}}(P^0 + P^3)$, and the 3-direction is singled out by the gauge choice $A^+ = 0$. $T^{++}$ is normalized much like $T^{00}$,

$$\langle P|T^{++}|P\rangle = 2{P^+}^2. \tag{2.7}$$

† $T^{\mu\nu}$ is ambiguous up to certain total derivatives, but these do not change the arguments presented here.

Unlike $T^{00}$, $T^{++}$ simplifies dramatically in $A^+ = 0$ gauge because of the simplification of $D^+$ and $F^{+\alpha}$,

$$\begin{aligned} D^+ &= \partial^+ - igA^+ \to \partial^+ \\ F^{+\alpha} &= \partial^+ A^\alpha - \partial^\alpha A^+ + g[A^+, A^\alpha] \to \partial^+ A^\alpha \,. \end{aligned} \tag{2.8}$$

As a result $T^{++}$ is quadratic in the fundamental dynamical variables, $q_+$ and $\vec{A}_\perp$ and all interactions disappear,

$$T^{++} = i q_+^\dagger \partial^+ q_+ + \mathrm{Tr}(\partial^+ \vec{A}_\perp)^2 \,. \tag{2.9}$$

The two terms give the contributions of quarks and gluons respectively to the total $P^+$. It is straightforward to relate each to an integral over a positive definite parton "momentum" distribution,

$$\begin{aligned} i q_+^\dagger \partial^+ q &\to \int dx\, x q(x) \\ (\partial^+ \vec{A}_\perp)^2 &\to \int dx\, x g(x) \end{aligned} \tag{2.10}$$

in which the parton probability density is weighted by the observable (in this case $x$) appropriate to the sum rule. Keeping track of renormalization scale dependence and kinematic factors of $P^+$, one obtains the standard "Momentum" Sum Rule,

$$1 = \int_0^1 dx\, x \left\{ q(x, Q^2) + g(x, Q^2) \right\} \tag{2.11}$$

The lessons learned from this exercise generalize to the more difficult case of angular momentum:

- The time-components of the tensor densities associated with space time symmetries do not yield sum rules. Interactions do not drop out. They yield relations which are difficult to interpret because quark and gluon contributions do not separate. Individual terms are not related to integrals over parton distributions.

- The +-components of the same tensor densities do yield useful sum rules, which have a parton interpretation in $A^+ = 0$ gauge. Interactions drop out. Each term can be represented as an integral over a parton distribution weighted by the appropriate observable quantity.

### (*b*) *Sum rules for angular momentum*

The situation for angular momentum is not satisfactory. The time-component analysis yields a relation, some of whose ingredients can be measured (in principle) in deeply virtual Compton scattering. But it has no place for a separately gauge invariant gluon spin and orbital angular momentum, no clean separation between quark and gluon contributions, and no relation to quark or gluon $x$ distributions. The + component analysis yields a classic sum rule with separate quark and gluon spin and orbital angular momentum contributions, each gauge invariant, each related to a parton distribution and each free from interaction terms. Unfortunately,

there does not seem to be a way to measure the terms in this otherwise perfectly satisfactory the sum rule.

The tensor density associated with rotations and boosts is a three component tensor antisymmetric in the last two indices, $M^{\mu\nu\lambda}$. To extract a sum rule, we polarize the nucleon along the 3-direction in its rest frame and set $\nu = 1, \lambda = 2$ in order to select rotations about this direction. The matrix elements of $M^{012}$ and $M^{+12}$ are both normalized in terms of the nucleon's momentum ($P^\mu = (M, 0, 0, 0)$) and spin ($S^\mu = (0, 0, 0, M)$) (Jaffe & Manohar 1990).

First consider the time component, $M^{012}$ (Ji 1997),

$$M^{012} = \frac{i}{2} q^\dagger (\vec{x} \times \vec{D})^3 q + \tfrac{1}{2} q^\dagger \sigma^3 q + 2\,\mathrm{Tr}\, E^j (\vec{x} \times i\vec{D})^3 A^j + \mathrm{Tr}(\vec{E} \times \vec{A})^3 . \tag{2.12}$$

The four terms look like the generators of rotations (about the 3-axis) for quark orbital, quark spin, gluon orbital, and gluon spin angular momentum respectively. Taking the matrix element in a nucleon state at rest, one obtains

$$\tfrac{1}{2} = \hat{L}_q + \tfrac{1}{2}\Sigma + \hat{L}_g + \Delta\hat{G} . \tag{2.13}$$

There are problems, however. There are no parton representations for $\hat{L}_g$, $\hat{L}_q$, or $\Delta\hat{G}$, so it is not a sum rule in the classic sense. $\Sigma$ is the integral of the helicity weighted quark distribution, but $\Delta\hat{G}$ is not the integral of the helicity weighted gluon distribution. Interactions prevent a clean separation into quark and gluon contributions as they did for $T^{00}$. And worse still, $\hat{L}_g$ and $\Delta\hat{G}$ are not separately gauge invariant, so only the sum $\hat{J}_g = \hat{L}_g + \Delta\hat{G}$ is physically meaningful.

The most important feature of the relation, equation (2.13), is the result derived by Ji, that $\hat{J}_q = \hat{L}_q + \frac{1}{2}\Sigma$ and $\hat{J}_g$ can, in principle, be measured in deeply virtual Compton scattering (Ji 1997).

Turning to the $+$-component sum rule, we find a much simpler form,

$$M^{+12} = \tfrac{1}{2} q_+^\dagger (\vec{x} \times \vec{i\partial})^3 q_+ + \tfrac{1}{2} q_+^\dagger \gamma_5 q_+ + 2\,\mathrm{Tr}\, F^{+j} (\vec{x} \times \vec{i\partial}) A^j + \mathrm{Tr}\, \epsilon^{+-ij} F^{+i} A^j \tag{2.14}$$

in $A^+ = 0$ gauge.† The four terms in $M^{+12}$ correspond respectively to quark orbital angular momentum, quark spin, gluon orbital angular momentum, and gluon spin, all about the 3-axis. Each is separately gauge invariant‡ and involves only the "good", i.e., dynamically independent, degrees of freedom, $q_+$ and $\vec{A}_\perp$. Each is a generator of the appropriate symmetry transformation in light-front field theory. The resulting sum rule,

$$\tfrac{1}{2} = L_q + \tfrac{1}{2}\Sigma + L_g + \Delta G \tag{2.15}$$

is a classic deep inelastic sum rule. It can be written as an integral over $x$-distributions

$$\tfrac{1}{2} = \int_0^1 dx\, \left\{ L_q(x, Q^2) + \tfrac{1}{2}\Delta q(x, Q^2) + L_g(x, Q^2) + \Delta G(x, Q^2) \right\} \tag{2.16}$$

† This gauge condition must be supplemented by the additional condition that the gauge fields vanish fast enough at infinity.

‡ Note however, that in any gauge other than $A^+ = 0$, the operators are nonlocal and appear to be interaction dependent. The same happens to the simple operators involved in the momentum sum rule, equation (2.9)

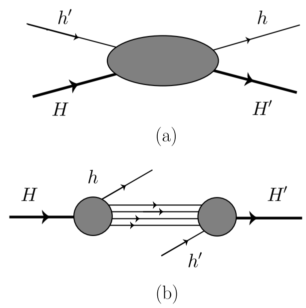


Figure 1. Quark hadron forward scattering. Quark helicities are labelled $h$ and $h'$; hadron helicities are $H$ and $H'$. (a) Full scattering amplitude; (b) u-channel discontinuity which gives the quark distribution function in DIS.

where each term is an interaction independent, gauge invariant, integral over a partonic density associated with the appropriate symmetry generator.

Satisfying though equations (2.15) and (2.16) may be from a theoretical point of view, they are quite useless unless someone finds a way to measure the two new terms $L_q$ and $L_g$.

## 3. The Parton Distribution for Transversity

There are many different ways of looking at parton distributions that help us understand different aspects of their physical significance. The transversity is most easily defined in the old fashioned parton model at infinite momentum, but its properties become clearer if we also look at it in the language of helicity amplitudes. First, in the parton model: consider a nucleon moving with (infinite) momentum in the $\hat{e}_3$-direction, but polarized along one of the directions transverse to $\hat{e}_3$. The transversity, $\delta q_a(x, Q^2)$, counts the quarks of (flavor $a$ and) momentum fraction $x$ polarized parallel to the nucleon minus the number antiparallel. Together, the unpolarized quark distribution, $q_a(x, Q^2)$, the quark helicity distribution, $\Delta q_a(x, Q^2)$, and the transversity provide a complete description of the quark spin in deep inelastic processes at leading twist (Jaffe 1996). If quarks moved nonrelativistically in the nucleon, $\delta q$ and $\Delta q$ would be identical, since rotations and Euclidean boosts commute and a series of boosts and rotations can convert a longitudinally polarized nucleon into a transversely polarized nucleon at infinite momentum. So the difference between the transversity and helicity distributions reflects the relativistic character of quark motion in the nucleon. There are other important differences between transversity and helicity. For example, quark and gluon helicity distributions ($\Delta q$ and $\Delta g$) mix under $Q^2$-evolution. There is no gluon analog of transversity in the nucleon, so $\delta q$ evolves without mixing, like a "non-singlet" distribution function.

Transversity is set apart from other parton distributions by its chiral transformation properties. A leading twist quark distribution can be viewed as a discontinuity in a quark-nucleon forward scattering amplitude labelled by the helicities of the external lines. This is the lower part of the standard "handbag diagram" shown in figure 1(a).

Conservation of angular momentum, parity and time reversal leave only three independent helicity amplitudes, $\mathcal{A}_{++,++}$, $\mathcal{A}_{+-,+-}$, and $\mathcal{A}_{+-,-+}$, where the sub-

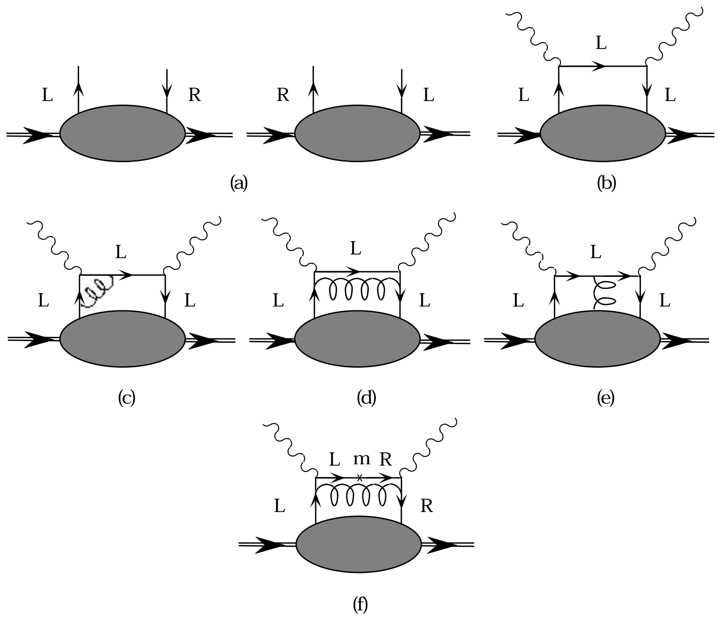


Figure 2. Chirality in deep inelastic scattering: (a) Chirally odd contributions to $h_1(x)$; (b)–(e) chirally even contributions to deep inelastic scattering (plus $L \leftrightarrow R$ for electromagnetic currents); (f) chirality flip by mass insertion.

scripts denote the quark and nucleon helicities as labelled in the figure. At leading twist, quark helicity and chirality are identical. The spin average ($q$) and helicity ($\Delta q$) distributions involve $\mathcal{A}_{++,++}$, $\mathcal{A}_{+-,+-}$, which preserve quark helicity, but the transversity corresponds to helicity (and therefore chirality) flip, $\mathcal{A}_{+-,-+}$. This is a simple consequence of quantum mechanics. The two states of transverse polarization can be written as superpositions of helicity eigenstates: $|\genfrac{}{}{0pt}{}{\perp}{\top}\rangle = \frac{1}{\sqrt{2}}(|+\rangle \pm |-\rangle)$; the cross section with transverse polarization has the form $d\sigma_{\genfrac{}{}{0pt}{}{\perp}{\top}} \propto \langle \genfrac{}{}{0pt}{}{\perp}{\top}|\cdots|\genfrac{}{}{0pt}{}{\perp}{\top}\rangle$; so the difference of cross sections is proportional to helicity flip, $d\sigma_\perp - d\sigma_\top \propto \langle +|\cdots|-\rangle + \langle -|\cdots|+\rangle$. For this reason, the transversity distribution called "chiral-odd", in contrast to the "chiral-even" distributions, $q$ and $\Delta q$.

Quark chirality is conserved at all QCD and electroweak vertices, however quark chirality can flip in distribution functions because they probe the soft regime where chiral symmetry is dynamically broken in QCD. This is another reason to be interested in transversity—it probes dynamical chiral symmetry breaking, an incompletely understood aspect of QCD.

Because all hard QCD and electroweak processes preserve chirality, transversity is difficult to measure. It decouples from inclusive DIS and most other familiar deep inelastic processes. The argument is made graphically in figure 2. This makes it difficult to observe transversity. Some process must flip the quark chirality a second time. The classic example, where transversity was discovered by Ralston and Soper, is transversely polarized Drell-Yan production of muon pairs: $\vec{p}_\perp \vec{p}_\perp \to \mu^+\mu^- X$, which is shown diagrammatically in figure 3. Chirality is flipped in both soft distribution functions and the cross section is proportional to $\delta q(x_1, Q^2) \times \delta\bar{q}(x_2, Q^2)$.

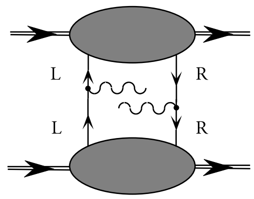


Figure 3. Chirality in Drell-Yan (plus $L \leftrightarrow R$) production of lepton pairs.

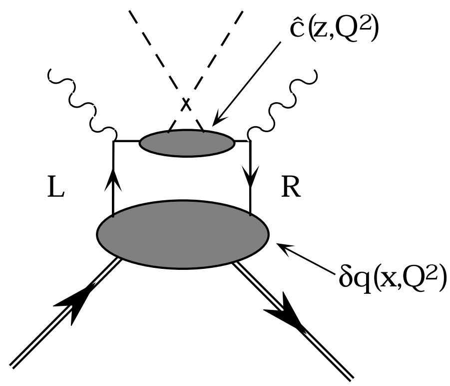


Figure 4. Single particle inclusive scattering $ep \to ehX$. The chiral-odd fragmentation function, denoted $\hat{c}$ in the figure, compensates for the chirality flip in the distribution function, denoted $\delta q$ in the figure.

Transversity would not decouple from deep inelastic scattering if some electroweak vertex would flip chirality. Unfortunately (and accidentally from the point of view of QCD) all photon, $W^{\pm}$ and $Z^0$ couplings preserve chirality. Quark-Higgs couplings violate chirality but are too weak to be of interest. Quark mass insertions flip chirality (see figure 1(f)), and indeed a careful analysis reveals effects proportional to $m\delta q(x, Q^2)/\sqrt{Q^2}$ in inclusive DIS with a transversely polarized target. However the $u$, $d$, and $s$ quarks, which are common in the nucleon, are too light to give significant sensitivity to $\delta q$.

What is needed is an insertion that flips chirality without introducing a $1/\sqrt{Q^2}$ suppression. Fortunately there are several candidates in the form of chiral-odd *fragmentation functions* which describe the production of hadrons in the current fragmentation function of deep inelastic scattering. A generic example is shown in figure 4. Several candidates have been proposed and studied in some detail.

- $\delta\hat{q}_a(z, Q^2)$, the transverse, spin-dependent fragmentation function. This is the analog in fragmentation of transversity, and describes the fragmentation of a transversely polarized quark into a transversely polarized hadron with momentum fraction $z$ (Jaffe & Ji 1993, Boer 2000). To access $\delta\hat{q}$, it is necessary to measure the spin of a particle in the final state of DIS. In practice this limits the application to production of a $\Lambda$ hyperon—the only particle whose spin is easy to measure through its parity violating decay.

- $\delta\hat{q}_I(z, m^2, Q^2)$, the two pion interference fragmentation function (Collins et al.

1994, Collins & Ladinsky 1994, Jaffe et al. 1998). This describes the fragmentation of a transversely polarized quark into a pair of pions whose orbital angular momentum is correlated with the quark spin. This requires measurement of two pions in the final state. It may be quite useful, especially in polarized collider experiments. I will not discuss it further here.

- $\hat{c}(z,Q^2)$, the single particle azimuthal asymmetry fragmentation function. This function, first discussed by Collins et al. (1994). describes the azimuthal distribution of pions about the axis defined by the struck quark's momentum in deep inelastic scattering.

All three of these fragmentation functions are chiral-odd and therefore produce experimental signatures sensitive to the transversity distribution in the target nucleon. Each may play an important role in future experiments aimed at probing the nucleon's transversity. Recently HERMES has announced observation of a spin asymmetry which seems to be associated with the Collins function, $c(z,Q^2)$. So although all three deserve discussion, I will spend the rest of my time on the Collins function and the HERMES asymmetry.

### (*a*) *The Collins Fragmentation Function*

The standard description of fragmentation without polarization requires a single fragmentation function usually called $D_h(z)$. It gives the probability that a quark will fragment into a hadron, $h$, with longitudinal momentum fraction $z$.† The transverse momentum of $h$ relative to the quark is integrated out. If the transverse momentum, $\vec{p}_\perp$, is observed, then it is possible to construct distributions weighted by geometric factors. For instance,

$$c(z) \quad \propto \quad \int d^2p_\perp D_h(z,\vec{p}_\perp)\cos\chi\,,$$

where, for comparison,

$$D(z) \quad \propto \quad \int d^2p_\perp D_h(z,\vec{p}_\perp)\,, \tag{3.1}$$

Here $D_h(z,\vec{p}_\perp)$ is the probability for the quark to fragment into hadron $h$ with momentum fraction $z$ and transverse momentum $\vec{p}_\perp$. $\chi$ is the angle between $\vec{p}_\perp$ and some vector, $\vec{w}$, defined by the initial state. Since we don't know the direction of the quark's momentum exactly, the transverse momentum of the hadron, $\vec{p}_\perp$, is defined relative to some large, externally determined momentum, such as the momentum of the virtual photon, $\vec{q}$, in DIS.

How can c(z) figure in deep inelastic scattering? The trick is to find a vector, $\vec{w}$, relative to which $\chi$ can be defined. If the target is polarized, it is possible to define $\vec{w}$ by taking the cross product of the target spin, $\vec{s}$, with either the initial or final electron's momentum ($\vec{k}$ or $\vec{k}'$) depending on the circumstances. Generically, then, the observable associated with $c(z)$ is $\cos\chi\propto\vec{k}\times\vec{s}\cdot\vec{p}$, where $\vec{p}$ is the momentum of the observed hadron in the final state. The situation is illustrated in figure 5 from Boer (1999). This observable is even under parity (because $\vec{s}$ is a pseudovector), but

† For simplicity I suppress the dependence of $D$ on the virtuality scale, $Q^2$ and the quark flavor label $a$.

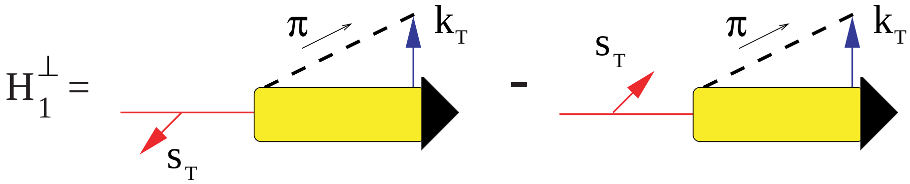


Figure 5. The Collins effect function $H_1^\perp$ signals different probabilities for $q(\pm\vec{S}_T) \to \pi(\vec{k}_T) + X$.

odd under time-reversal. This *does not* mean that it violates time reversal invariance. Instead it means that it will vanish unless there are final state interactions capable of generating a nontrivial phase in the DIS amplitude. This subtlety makes it hard to find a good model to estimate $c(z)$ because typical fragmentation models involve only tree graphs (if they involve quantum mechanics at all!) which are real.

The Collins fragmentation function, $c(z)$, may be interesting in itself, but it is much more interesting because it is chiral-odd and combines with the transversity distribution in the initial nucleon to produce an experimentally observable asymmetry sensitive to the transversity. Two specific cases figure in recent and soon-to-be-performed experiments.

(i) *Single particle inclusive DIS with a transversely polarized target:* $e\vec{p}_\perp \to e'\pi X$

If the target is transversely polarized (with respect to the initial electron momentum, $\vec{k}$), then $\vec{w} = \vec{k} \times \vec{s}$ defines a vector normal to the plane defined by the beam and the target spin. The transverse momentum of the produced hadron can be defined either with respect to the beam or the momentum transfer $\vec{q}$—the difference in higher order in $1/Q$. $\cos\chi$ is defined by $\cos\chi = \vec{p}_\perp \cdot \vec{w}/|\vec{p}_\perp||\vec{w}|$. The kinematics are particularly simple in this case (transverse spin). Experimenters prefer to think of the effect in terms of the angle ($\phi$) between two planes: Plane 1 is defined by the virtual photon and the target spin, and Plane 2 is defined by the virtual photon and the transverse momentum of the produced hadron. Then $\sin\phi = \cos\chi$ and the effect is known as a "$\sin\phi$" asymmetry. When the cross section is weighted by $\sin\phi$ the result is

$$\frac{d\Delta\sigma_\perp}{dx\, dy\, dz} = \frac{2\alpha^2}{Q^2}\sum_a e_a^2 \delta q_a(x) c_a(z) \tag{3.2}$$

where $y = E - E'/E$, and $\Delta\sigma$ is the difference of cross sections with target spin reversed.† This is a leading twist effect, which scales (modulo logarithms of $Q^2$) in the deep inelastic limit. If $c(z)$ is not too small, it is will become the "classic" way to measure the nucleon's transversity distributions.

No experimental group has yet measured hadron production in deep inelastic scattering from a transversely polarized target, so there is no data on $\Delta\sigma_\perp$. HERMES at DESY intend to take data under these conditions in the next run. One

† In principle this reversal is superfluous because the $\sin\phi$ asymmetry must be odd under reversal and the rest of the cross section must be even. However, it helps experimenters to reduce systematic errors.

reason for this was the observation of a $\sin\phi$ asymmetry with a *longitudinally* polarized target which HERMES announced last year (Airapetian 1999). It strongly suggests, but does not require, that $\Delta\sigma_\perp$ should be large.

(ii) *Single particle inclusive DIS with a longitudinally polarized target:* $e\vec{p}_\parallel \to e'\pi X$

The possibility of a $\sin\phi$ asymmetry is more subtle in this case and escaped theorists attention for a long time. Such an asymmetry was first pointed out by Oganessian et al.(1998). As $Q^2$ and $\nu$ go to $\infty$, the initial and final electron's momenta become parallel. If the target spin is parallel to $\vec{k}$, then it is impossible to construct a vector from $\vec{k}$ or $\vec{k}'$ and $\vec{s}$ in this limit. However $\vec{k}$ and $\vec{k}'$ are not exactly parallel, so $\vec{s}$ has a small component perpendicular to the virtual photon's momentum, $\vec{q} = \vec{k} - \vec{k}'$. The vector, $\vec{w}$, can be defined as $\vec{w} = \vec{k}' \times \vec{s}$, and the kinematic situation is shown in figure 6 from Airapetian (1999). This produces an

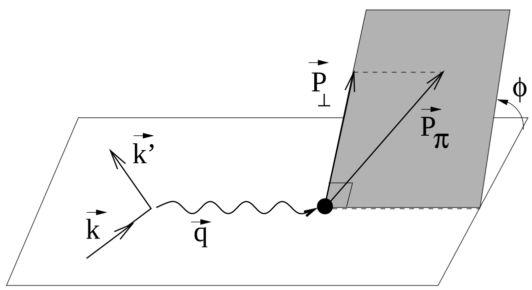


Figure 6. Kinematic planes for pion production in semi-inclusive deep-inelastic scattering.

asymmetry similar to the previous case, but weighted by $|\vec{s}_\perp| \propto 2Mx/Q$. Because this leading (twist two) effect is kinematically suppressed by $1/Q$, it is necessary to consider other, twist-three, effects which might be competitive. A careful analysis turns up a variety of twist-three effects, leading to a cross section of the form (Boer 1999, Oganessian et al. 1998),

$$\frac{d\Delta\sigma_\parallel}{dx\,dy\,dz} = \frac{2\alpha^2}{Q^2}\frac{2Mx}{Q}\sqrt{1-y}\sum_a e_a^2\left\{\delta q_a(x)c_a(z) + \frac{2-y}{1-y}h_{La}(x)c_a(z)\right\}, \tag{3.3}$$

where $h_L(x)$ is a longitudinal spin dependent, twist three distribution function analogous to $g_T$.

By far the most interesting thing about $\Delta\sigma_\parallel$ is that HERMES has seen such an asymmetry in their $\pi^+$ data. The HERMES data is shown in figure 7. They see no effect in their $\pi^-$ data. Because $u$ quarks predominate in the nucleon, because $e_u^2 = 4e_d^2$, and because $u \to \pi^+ \gg u \to \pi^-$, they expect no signal in $\pi^-$. They have not reported on $\pi^0$, where an asymmetry similar to $\pi^+$ would be expected (K. A. Oganessian 2000, personal communication).

If the HERMES result is confirmed, it demonstrates that the Collins fragmentation function is nonzero. Somehow the final state interactions between the observed pion and the other fragments of the nucleon suffice to generate a phase that survives

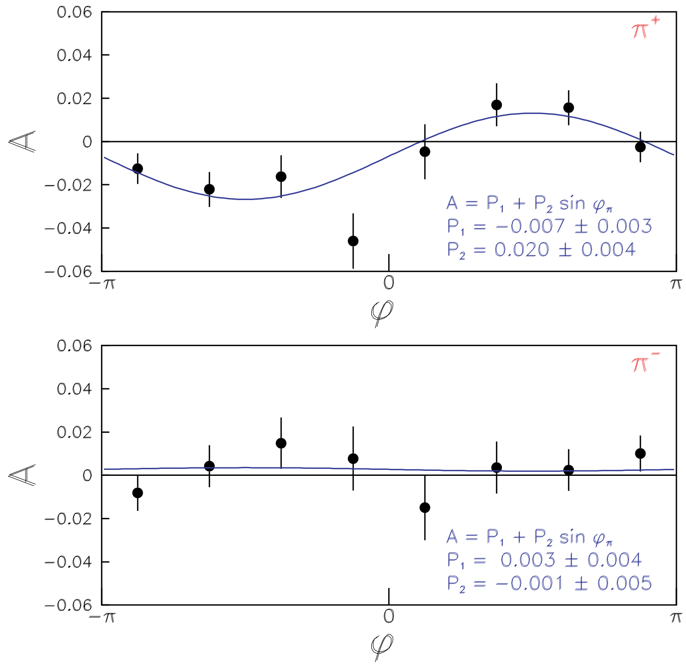


Figure 7. Azimuthal asymmetry ($\sin\phi$) distribution for $\pi+$ and $\pi^-$ production with a longitudinally polarized target at HERMES.

the sum over the other unobserved hadrons. Whatever its origin, a nonvanishing Collins function would be a great gift to the community interested in the transverse spin structure of the nucleon. It provides an unanticipated tool for extracting the nucleon's transversity from DIS experiments. The fact that HERMES has seen a robust (2–3%) asymmetry with a longitudinally polarized target suggests that they will see a large asymmetry with a transversely polarized target (unless the effect is entirely twist three—e.g., $h_L \gg \delta q$). This in turn will lead to the first measurements of the nucleon's transversity distribution and to new insight into the relativistic spin structure of confined states of quarks and gluons.

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